Problem Set 3 - LV 141.246 QISS - 16.4.2012

1. **Lindblad Equation**

   Consider a qubit
$$H_0 = \frac{\hbar\omega_0}{2}\left(|1\rangle\langle 1| - |0\rangle\langle 0|\right)$$
   with a classical driving field
$$H_\mathrm{d} = \frac{\hbar\omega_1}{2}\left(|1\rangle\langle 0|e^{i\omega t} + |0\rangle\langle 1|e^{-i\omega t}\right)$$
   Furthermore consider relaxation with the Lindblad operator
$$L_r = \sqrt{\gamma_r}|0\rangle\langle 1|$$
   and dephasing
$$L_\phi = \sqrt{\gamma_\phi}(|0\rangle\langle 0| - |1\rangle\langle 1|)$$
   Write the Lindblad equation in the basis
$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
   Show that you can write the four coupled ordinary differential equations in the form
$$\frac{\partial}{\partial t}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} = -i\omega_0\mathbf{U_0}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} - i\omega_d\mathbf{U}_\mathrm{d}\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} + \gamma_r\mathbf{L}_r\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix} + \gamma_p\mathbf{L}_p\begin{pmatrix} \rho_{00} \\ \rho_{01} \\ \rho_{10} \\ \rho_{11} \end{pmatrix}$$

2. **Qubit Simulation - Relaxation**

   One can numerically solve an ordinary differential equation in Matlab with the function `ode45`. One has to provide the derivative

```
function drho = Lindblad(t, rho)
global omega omega0 omega1 gammar gammap

dp = omega1*exp(i*omega*t);
dm = omega1*exp(-i*omega*t);
U=[  0      -dp      dm   0;...
   -dm -omega0       0  dm;...
    dp        0 omega0 -dp;...
     0       dp     -dm   0];

Lr = [0 0     0     1;...
      0 -1/2 0     0;...
```

```
        0 0     -1/2 0;...
        0 0     0    -1];

Lp = [0 0  0  0;...
      0 -2 0  0;...
      0 0  -2 0;...
      0 0  0  0];

drho = -i*U*rho+gammar*Lr*rho+gammap*Lp*rho;
```

Now solve the time evolution of the qubit state without driving starting in the state

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

```
global omega omega0 omega1 gammar gammap
gammap=0;gammar=0.1;omega0=1;omega1=0;omega=0;
[t,rho]=ode45(@Lindblad,[0 30],rho0);
```

Plot $\langle s_z \rangle$, $\mathrm{tr}(\rho)$ and $\mathrm{tr}(\rho^2)$ versus time.

3. Qubit Simulation - Dephasing

Now we add dephasing to the qubit

```
gammap=0.1; gammar=0.1; omega0=1; omega1=0; omega=0;
```

Plot $\langle s_x \rangle$, $\langle s_y \rangle$ and $\sqrt{\langle s_x \rangle^2 + \langle s_y \rangle^2}$ versus time. Comment your result.

4. Qubit Simulation - Rabi oscillation

Finally we add a oscillatory driving field causing Rabi cycles. Start in the ground state, i.e. $|0\rangle$

Plot $\langle s_z \rangle$ for a strong driving

```
gammap=0.1; gammar=0.1; omega0=1; omega1=2; omega=1;
```

and weak driving

```
gammap=0.1; gammar=0.1; omega0=1; omega1=0.5; omega=1;
```